Spin-Dependent Parton Distributions
of the Longitudinally Polarized Photon
Beyond the Leading Order

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Abstract

A next-to-leading order (NLO) QCD analysis of the spin-dependent parton distributions $\Delta f^\gamma(x, Q^2)$ of the longitudinally polarized photon and of its structure function $g_1^\gamma(x, Q^2)$ is performed within the framework of the radiative parton model. The important issues of a suitably chosen factorization scheme and related boundary conditions are discussed in detail. The typical effects of the NLO corrections are quantitatively studied for two very different conceivable scenarios for the NLO polarized parton distributions $\Delta f^\gamma(x, Q^2)$.

More accurate measurements of the nucleon's spin asymmetry $A_1^N(x, Q^2) \simeq g_1^N(x, Q^2)/F_1^N(x, Q^2)$ in polarized deep-inelastic scattering (DIS) [1], covering also a wider range in (x, Q^2) and providing results for different targets ($N = n, d$) as compared to early measurements of $A_1^p(x, Q^2)$ [2], have considerably improved our knowledge about the nucleon's spin structure in the past few years and also renewed the theoretical interest in this field. This is also due to the possibility to perform now a complete and consistent QCD analysis of polarized DIS in NLO, since the required spin-dependent two-loop splitting functions $\Delta P_{ij}^{(1)}$ have been calculated recently [3, 4]. A first such NLO analysis in the $\overline{\text{MS}}$ scheme has been presented in [5] based on the phenomenologically successful concept of the radiative parton model, i.e., the generation of parton distributions from a valence-like structure at some low-resolution scale μ, which had previously led, e.g., to the prediction [6] of the small-x rise of the unpolarized proton structure function $F_2^p(x, Q^2)$ as observed at HERA [7]. Subsequent NLO studies [8] have imposed different boundary conditions and/or factorization schemes.

The knowledge of the two-loop splitting functions $\Delta P_{ij}^{(1)}$ [3, 4] also offers the opportunity to perform a similar NLO QCD analysis of the spin-dependent parton content Δf^γ of the longitudinally (more precisely, circularly) polarized *photon* because the required two-loop photon-to-parton splitting functions $\Delta k_q^{(1)} \equiv \Delta P_{q\gamma}^{(1)}$ and $\Delta k_g^{(1)} \equiv \Delta P_{g\gamma}^{(1)}$ can be easily obtained from $\Delta P_{qg}^{(1)}$ and $\Delta P_{gg}^{(1)}$, respectively. Although such a study seems to be somewhat premature in view of the lack of *any* experimental information on Δf^γ up to now, interesting theoretical questions arise when going beyond the leading order. Apart from getting a feeling for the typical size of the NLO corrections it is moreover important to analyse the necessity (and feasibility) to introduce a suitable factorization scheme which overcomes expected problems with perturbative instabilities arising in the $\overline{\text{MS}}$ scheme in particular for large values of x. Such instabilities were found in the unpolarized case where they were eliminated [9] by absorbing the the 'direct-photon' contribution to F_2^γ into the NLO photonic quark distributions (DIS$_\gamma$ scheme). In this paper we will show that a similar procedure is also recommendable in the polarized case, where it works equally well.

Furthermore it is no longer inconceivable to longitudinally polarize also the proton beam at HERA [10]. At such high energies the polarized electron acts dominantly as a source of almost real (Weizsäcker-Williams) photons, thus measurements of double spin asymmetries in, e.g., the photoproduction of large-p_T jets can in principle reveal information on the parton content of the polarized photon in addition to that of the proton [11] through the presence of resolved-photon processes. In the corresponding situation with unpolarized beams this has been already extensively studied experimentally [7]. Future polarized linear e^+e^- colliders could serve to provide additional complementary information on Δf^γ [12] by measuring the spin-dependent photon structure function $g_1^\gamma(x, Q^2)$ or spin asymmetries in resolved two-photon reactions.

In the remainder of the paper we present all necessary ingredients for the two-loop evolution of the spin-dependent parton distributions of the photon and for the calculation of its structure function g_1^γ in NLO, analysing also the aforementioned theoretical questions. We will work within the framework of the radiative parton model since the corresponding analysis for the unpolarized photon [13] has again been phenomenologically very successful [7]. We will present two 'extreme' sets of polarized NLO distributions $\Delta f^\gamma(x, Q^2)$ following closely a previous LO analysis [14, 12].

Similarly to the purely hadronic case it is convenient to decompose the spin-dependent parton distributions $\Delta f^\gamma(x, Q^2)$ ($f = u$, d, s, g) of the longitudinally polarized photon into flavor non-singlet (NS) quark combinations Δq_{NS}^γ and the singlet (S) part $\Delta \vec{q}_S^\gamma \equiv \left(\begin{smallmatrix} \Delta \Sigma^\gamma \\ \Delta g^\gamma \end{smallmatrix} \right)$, where $\Delta \Sigma^\gamma \equiv \sum_f (\Delta f^\gamma + \Delta \bar{f}^\gamma)$ with f running over all relevant active quark flavors and Δg^γ denotes the polarized photonic gluon distribution. The so defined combinations $\Delta q_i^\gamma(x, Q^2)$ (i=NS, S) satisfy the well-known *inhomogeneous* evolution equations schematically given by[1]

$$\frac{d\Delta q_i^\gamma(x, Q^2)}{d \ln Q^2} = \Delta k_i(x, Q^2) + (\Delta P_i * \Delta q_i^\gamma)(x, Q^2) \ , \tag{1}$$

where the symbol $*$ denotes the usual convolution in Bjorken-x space which reduces, in Mellin-n space, to a simple product $\Delta P_i^n \Delta q_i^{\gamma, n}$ with the nth moment of a function $h(x, Q^2)$

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[1] We follow closely the notation adopted in the unpolarized case as presented in refs.[15] and [9].

being defined as

$$h^n(Q^2) \equiv \int_0^1 x^{n-1} h(x, Q^2) \, dx \quad . \tag{2}$$

The polarized photon-to-parton and parton-to-parton splitting functions, $\Delta k_i(x, Q^2)$ and $\Delta P_i(x, Q^2)$, respectively, in eq.(1) receive the following 1-loop (LO) and 2-loop (NLO) contributions (i=NS, S):

$$\Delta k_i(x, Q^2) = \frac{\alpha}{2\pi} \Delta k_i^{(0)}(x) + \frac{\alpha \alpha_s(Q^2)}{(2\pi)^2} \Delta k_i^{(1)}(x)$$

$$\Delta P_i(x, Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \Delta P_i^{(0)}(x) + \left(\frac{\alpha_s(Q^2)}{2\pi} \right)^2 \Delta P_i^{(1)}(x) \quad , \tag{3}$$

where $\alpha \simeq 1/137$ and the NLO running strong coupling is given by

$$\frac{\alpha_s(Q^2)}{4\pi} \simeq \frac{1}{\beta_0 \ln Q^2/\Lambda_{\overline{\text{MS}}}^2} - \frac{\beta_1}{\beta_0^3} \frac{\ln \ln Q^2/\Lambda_{\overline{\text{MS}}}^2}{\left(\ln Q^2/\Lambda_{\overline{\text{MS}}}^2 \right)^2} \tag{4}$$

with $\beta_0 = 11 - 2N_f/3$, $\beta_1 = 102 - 38N_f/3$, and N_f being the number of active flavors. In the singlet (S) case eq.(1) becomes, of course, a coupled 2×2 matrix equation where

$$\Delta \hat{P}_S^{(j)} \equiv \begin{bmatrix} \Delta P_{qq}^{(j)} & \Delta P_{qg}^{(j)} \\ \Delta P_{gq}^{(j)} & \Delta P_{gg}^{(j)} \end{bmatrix} \quad , \quad \Delta \vec{k}_S^{(j)} = \begin{pmatrix} \Delta k_q^{(j)} \\ \Delta k_g^{(j)} \end{pmatrix} \tag{5}$$

in eq.(3) with $j = 0, 1$. The hadronic polarized splitting functions $\Delta P_{ff'}^{(j)}$ can be found in [3, 4] and apart from obvious NS and S charge factors, $\langle e^4 \rangle - \langle e^2 \rangle^2$ and $\langle e^2 \rangle$, respectively, where $\langle e^k \rangle \equiv N_f^{-1} \sum_{i=1}^{N_f} e_{q_i}^k$, the spin-dependent photon-to-parton splitting function $\Delta k_q^{(0)}$ can be obtained from $\Delta P_{qg}^{(0)}$ by multiplying it with $N_f N_C/T_F$ where $N_C = 3$ and $T_F = N_f/2$; similarly the NLO quantities $\Delta k_q^{(1)}$ and $\Delta k_g^{(1)}$ correspond to the $C_F T_F$ terms of $\Delta P_{qg}^{(1)}$ and $\Delta P_{gg}^{(1)}$, respectively, multiplied by $N_f N_C/T_F$:[2]

$$\Delta k_{NS}^{(j)}(x) = N_f N_C (\langle e^4 \rangle - \langle e^2 \rangle^2) \Delta \kappa^{(j)}(x) , \quad \Delta k_q^{(j)}(x) = N_f N_C \langle e^2 \rangle \Delta \kappa^{(j)}(x)$$

$$\Delta \kappa^{(0)}(x) = 2 \left[x^2 - (1-x)^2 \right]$$

$$\Delta \kappa^{(1)}(x) = C_F \left[-9 \ln x + 8(1-x) \ln(1-x) + 27x - 22 \right.$$

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[2] Note that $\Delta k_g^{(0)} = 0$ due to the missing photon-gluon coupling in lowest order. Furthermore, there is a subtlety in deriving $\Delta k_g^{(1)}$ from $\Delta P_{gg}^{(1)}$ because the latter splitting function is a diagonal quantity and hence contains $\delta(1-x)$ terms originating from gluon self-energy contributions which have to be omitted in $\Delta k_g^{(1)}$ [9, 16].

$$+ \ \frac{1}{2}\left\{ \ln^2 x + 2\ln^2(1-x) - 4\ln x \ln(1-x) - \frac{2}{3}\pi^2 \right\}\Delta\kappa^{(0)}(x)\right]$$

$$\Delta k_g^{(0)}(x) \ = \ 0$$

$$\Delta k_g^{(1)}(x) \ = \ N_f N_C \langle e^2 \rangle C_F \left[-2(1+x)\ln^2 x + 2(x-5)\ln x - 10(1-x) \right] \quad , \qquad (6)$$

where $C_F = 4/3$.

The evolution equations (1) are most conveniently solved directly in Mellin-n space where the solutions can be given analytically and one can easily keep track of the contributions stemming from different powers of α_s in order to avoid terms beyond the order considered. Taking, according to eq.(2), the nth moment of eq.(1) the various convolutions simply factorize and the required moments of the photonic inhomogeneous LO and NLO Δk terms in eqs.(1,3,5,6) are given by

$$\Delta k_{NS}^{(j)n} \ = \ N_f N_C(\langle e^4 \rangle - \langle e^2 \rangle^2)\Delta\kappa^{(j)n} \ , \quad \Delta k_q^{(j)n} = N_f N_C \langle e^2 \rangle \Delta\kappa^{(j)n}$$

$$\Delta\kappa^{(0)n} \ = \ 2\frac{n-1}{n(n+1)}$$

$$\Delta\kappa^{(1)n} \ = \ C_F \Delta\kappa^{(0)n}\left[S_1(n)^2 - S_2(n) - \frac{2}{n}S_1(n) + \frac{5n^4 + 10n^3 - n + 2}{2n^2(n+1)^2} \right]$$

$$\Delta k_g^{(0)n} \ = \ 0$$

$$\Delta k_g^{(1)n} \ = \ N_f N_C \langle e^2 \rangle C_F \left[-2\frac{(n-1)(n+2)(n^2-n-1)}{n^3(n+1)^3} \right] \quad . \qquad (7)$$

with $S_k(n) \equiv \sum_{j=1}^{n} j^{-k}$. The Mellin moments $\Delta P_{ff'}^{(j)n}$ of the 1- and 2-loop hadronic splitting functions can be found in [5][3] in a form appropriate for a straightforward analytic continuation in n (also given in [5]) which is required for a numerical Mellin inversion back into x-space. The solution of eq.(1) can be decomposed into a 'pointlike' (inhomogeneous[4]) and a 'hadronic' (homogeneous) part, i.e.,

$$\Delta q_i^{\gamma,n}(Q^2) = \Delta q_{i,PL}^{\gamma,n}(Q^2) + \Delta q_{i,had}^{\gamma,n}(Q^2) \qquad (8)$$

($i =$ NS, S) and can be found in [9] (with the obvious replacements of all unpolarized quantities like, e.g., $k_i^{(1)n}$, by the corresponding polarized ones, e.g., $\Delta k_i^{(1)n}$); they need not

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[3]Note that the nth moments of the hadronic splitting functions $\Delta P_{ff'}^{(j)n}$ ($j = 0, 1$) and the anomalous dimensions $\Delta\gamma_{ff'}^{(j)n}$ as given in [3, 5] are related through $\Delta P_{ff'}^{(0)n} = -\frac{1}{4}\Delta\gamma_{ff'}^{(0)n}$ and $\Delta P_{ff'}^{(1)n} = -\frac{1}{8}\Delta\gamma_{ff'}^{(1)n}$.

[4]By definition, we choose the pointlike part to satisfy $\Delta q_{i,PL}^{\gamma,n}(\mu^2) = 0$ ($i =$NS,S) at the input scale μ.

be repeated here. Having solved the evolution equations (1) for $\Delta q_{NS}^{\gamma,n}(Q^2)$, $\Delta\Sigma^{\gamma,n}(Q^2)$, and $\Delta g^{\gamma,n}(Q^2)$ one finally obtains the desired photonic parton distributions $\Delta f^{\gamma,n}(Q^2)$ ($f = u,\, d,\, s,\, g$) by a straightforward flavor decomposition.

In moment-n space the NLO expression for the spin-dependent photon structure function g_1^γ is given by

$$
\begin{aligned}
g_1^{\gamma,n}(Q^2) \;=\; & \frac{1}{2}\sum_{f=u,d,s} e_f^2 \left\{ \Delta f^{\gamma,n}(Q^2) + \Delta\bar{f}^{\gamma,n}(Q^2) \right. \\
& + \left. \frac{\alpha_s(Q^2)}{2\pi}\left[\Delta C_q^n \left(\Delta f^{\gamma,n}(Q^2) + \Delta\bar{f}^{\gamma,n}(Q^2) \right) + \frac{1}{N_f}\Delta C_g^n \Delta g^{\gamma,n}(Q^2) \right] \right\} \\
& + \frac{1}{2} N_f N_C \langle e^4 \rangle \frac{\alpha}{2\pi}\Delta C_\gamma^n
\end{aligned}
\tag{9}
$$

with the usual hadronic spin-dependent Wilson coefficients ΔC_q^n and ΔC_g^n which in the conventional $\overline{\text{MS}}$ scheme can be found, e.g., in ref.[5]. The photonic coefficient ΔC_γ^n can be easily derived from ΔC_g^n and is in the $\overline{\text{MS}}$ scheme given by:

$$
\Delta C_\gamma^n = \frac{1}{T_F}\Delta C_g^n = 2\left[-\frac{n-1}{n(n+1)}\left(S_1(n) + 1 \right) - \frac{1}{n^2} + \frac{2}{n(n+1)} \right]
\tag{10}
$$

corresponding to the x-space expression

$$
\Delta C_\gamma(x) = 2\left[(2x-1)\left(\ln\frac{1-x}{x} - 1 \right) + 2(1-x) \right] \quad .
\tag{11}
$$

We note that the LO expression for g_1^γ is entailed in the above formula (9) by simply dropping all NLO terms, i.e., all ΔC_i^n ($i = q,\, g,\, \gamma$). For what follows it is convenient to introduce the decomposition of $g_1^{\gamma,n}(Q^2)$ into a pointlike and a hadronic part, analogously to eq.(8):

$$
g_1^{\gamma,n}(Q^2) \equiv g_{1,PL}^{\gamma,n}(Q^2) + g_{1,had}^{\gamma,n}(Q^2) \, ,
\tag{12}
$$

where $g_{1,PL}^{\gamma,n}(Q^2)$ is obtained from eq.(9) by taking only $\Delta f^{\gamma,n}(Q^2) = \Delta f_{PL}^{\gamma,n}(Q^2)$ with $\Delta f_{PL}^{\gamma,n}(Q^2)$ as defined in (8). Conversely, for $g_{1,had}^{\gamma,n}(Q^2)$ one uses the $\Delta f_{had}^{\gamma,n}(Q^2)$ of (8), and one obviously has to omit the ΔC_γ^n term in (9) in this case.

The desired x-space expressions for $\Delta f^\gamma(x, Q^2)$ and $g_1^\gamma(x, Q^2)$ can be easily obtained from the above given n-space expressions $\Delta f^{\gamma,n}(Q^2)$ and $g_1^{\gamma,n}(Q^2)$, respectively, by performing a standard numerical Mellin inversion.

The solutions for $\Delta f^{\gamma,n}(Q^2)$ $(\Delta f^\gamma(x, Q^2))$ depend on the up to now unspecified hadronic input distributions at the input scale $Q^2 = \mu^2$, i.e., on the boundary conditions for the hadronic pieces $\Delta f^{\gamma,n}_{had}$ in (8) which one would intuitively relate to some model inspired by vector meson dominance (VMD). On the other hand, beyond LO both the 'pointlike' as well as the 'hadronic' pieces in (8) depend on the factorization scheme chosen, and it is a priori not clear in which type of factorization schemes it actually makes sense to impose a pure VMD hadronic input. Indeed, in the unpolarized case it was observed that [9] the $\ln(1-x)$ term in the photonic coefficient function $C_{2,\gamma}(x)$ for F_2^γ, which becomes negative and divergent for $x \to 1$, drives the pointlike part of $F_2^\gamma(x, Q^2)$ in the $\overline{\text{MS}}$ scheme to large negative values as $x \to 1$, leading to a strong difference between the LO and the NLO results for $F^\gamma_{2,PL}$ in the large-x region. As illustrated in Fig.1, a very similar thing happens in the polarized case: Here it is the $\ln(1-x)$ term in the polarized photonic coefficient function $\Delta C_\gamma(x)$ (see eq.(11)) for g_1^γ that causes large negative values of the pointlike part of $g_1^\gamma(x, Q^2)$ in the $\overline{\text{MS}}$ scheme as $x \to 1$, strongly differing from the corresponding LO result also shown in Fig.1. Clearly, the addition of a VMD-inspired hadronic part $\Delta f^{\gamma,n}_{had}(Q^2)$ cannot be sufficient to cure this observed instability of $g^\gamma_{1,PL}$ in the large-x region since any VMD input vanishes as $x \to 1$. Instead, as in the unpolarized case, an appropriately adjusted ('fine tuned') non-VMD hadronic NLO input would be required in the $\overline{\text{MS}}$ scheme, substantially differing from the LO one, as the only means of avoiding unwanted and physically not acceptable perturbative instabilities for physical quantities like $g_1^\gamma(x, Q^2)$.

In the unpolarized case the so-called DIS$_\gamma$ scheme [9] was introduced to avoid such 'inconsistencies' by absorbing the photonic Wilson coefficient for F_2^γ into the photonic quark distributions. Analogously, one expects that a similar procedure for the coefficient ΔC_γ for g_1^γ cures the problem observed for $g^\gamma_{1,PL}$ in the $\overline{\text{MS}}$ scheme. This redefinition of the polarized photonic quark distributions implies, of course, also a transformation of the NLO photon-to-parton splitting functions $\Delta k_i^{(1)}$ due to the requirement that the physical quantity g_1^γ has to be scheme independent. In the polarized case the transformation to the DIS$_\gamma$ scheme reads

$$\Delta C_\gamma^n \to \Delta C_\gamma^n + \delta \Delta C_\gamma^n \ , \tag{13}$$

where $\delta\Delta C_\gamma^n = -\Delta C_\gamma^n$. This implies for the $\Delta k_i^{(1)n}$ (i =NS,S) in eq.(7) that $\Delta k_i^{(1)n} \to \Delta k_i^{(1)n} + \delta\Delta k_i^{(1)n}$ with [9]

$$
\begin{aligned}
\delta\Delta k_{NS}^{(1)n} &= -N_f N_C(\langle e^4\rangle - \langle e^2\rangle^2)\Delta P_{qq}^{(0)n}\Delta C_\gamma^n \\
\delta\Delta \vec{k}_S^{(1)n} &= \begin{pmatrix} \delta\Delta k_q^{(1)n} \\ \delta\Delta k_g^{(1)n} \end{pmatrix} = -N_f N_C\langle e^2\rangle \begin{pmatrix} \Delta P_{qq}^{(0)n}\Delta C_\gamma^n \\ \Delta P_{gq}^{(0)n}\Delta C_\gamma^n \end{pmatrix} \quad .
\end{aligned}
\tag{14}
$$

It should be emphasized that all hadronic quantities, in particular ΔC_q^n and ΔC_g^n, are unaffected by this kind of scheme transformation. We remark that if one chooses to solve the evolution equations for the DIS_γ polarized photonic parton distributions $\Delta f^\gamma(x, Q^2)$ directly in x-space by a (cumbersome) numerical iterative procedure the Mellin inverse of $\delta\Delta k_i^{(1)n}$ in eq.(14) is explicitly needed. Using standard integrals [17] and [9]

$$
\int_0^1 dx\, x^{n-1}\left[\ln^2(1-x) - \ln x \ln(1-x) - \text{Li}_2(x)\right] = \frac{1}{n}[S_1(n)]^2
\tag{15}
$$

one obtains for $\Delta\kappa_q^n \equiv \Delta P_{qq}^{(0)n}\Delta C_\gamma^n$

$$
\begin{aligned}
\Delta\kappa_q(x) = C_F\Big[&-7 + 4x + (-5 + 8x)\ln x + (15 - 16x)\ln(1-x) \\
&+ (2x-1)\left[4\ln^2(1-x) - 4\ln(1-x)\ln x + \ln^2 x + 2\text{Li}_2(x) - \pi^2\right]\Big] , \quad (16)
\end{aligned}
$$

and the inverse of $\Delta\kappa_g^n \equiv \Delta P_{gq}^{(0)n}\Delta C_\gamma^n$ reads

$$
\begin{aligned}
\Delta\kappa_g(x) = 2C_F\Big[&-12(1-x) + (-7+x)\ln x - (1+x)\ln^2 x + (1+x)\frac{\pi^2}{3} \\
&+ 5(1-x)\ln(1-x) - 2(1+x)\text{Li}_2(x)\Big] \quad .
\end{aligned}
\tag{17}
$$

Inspecting eqs.(6),(14),(16),(17) one finds that the transformation to the DIS_γ scheme, besides curing the instabilities at $x \to 1$, also eliminates all terms $\sim \ln^2 x$ from the polarized NLO photon-to-parton splitting functions $\Delta k_i^{(1)}(x)$ (i =NS,S), i.e., removes the $\overline{\text{MS}}$ terms leading for $x \to 0$ (for corresponding observations in the unpolarized case see [18, 19]).

The result for $g_{1,PL}^\gamma$ after the transformation to the DIS_γ scheme is also shown in Fig.1. The similarity between the NLO (DIS_γ) and the LO curves strongly suggests that it is indeed recommendable also in the polarized case to work in the DIS_γ scheme. We

note that it turns out, however, that the resulting $g_{1,PL}^{\gamma}$ slightly exceeds the pointlike part of the unpolarized photon structure function $F_{1,PL}^{\gamma}$ in the vicinity of $x \sim 0.6$, thus making a violation of the fundamental positivity constraint $|g_1^{\gamma}| \leq F_1^{\gamma}$ imminent there. The underlying reason for this feature is not a defect of the DIS_{γ} scheme as such, but resides in the fact that in the unpolarized case the DIS_{γ} scheme was formulated [9] in terms of (the only measured structure function) F_2^{γ}, and *not* F_1^{γ}. The difference between the unpolarized photonic coefficient functions C_1^{γ} and C_2^{γ} (for F_1^{γ} and F_2^{γ}, respectively) decreases $F_{1,PL}^{\gamma}$ with respect to $F_{2,PL}^{\gamma}/2x$, which explains the above effect. The problem could be straightforwardly resolved by repeating the analysis of [9, 13] in a modified DIS_{γ} scheme for which one would choose to absorb C_1^{γ} rather than C_2^{γ} into the unpolarized NLO photonic quark densities. This is clearly beyond the scope of this paper. We mention in this context that in the unpolarized case also an alternative factorization scheme was suggested [20] for which only the 'process independent' part of the photonic Wilson coefficient for F_2^{γ} is absorbed into the photonic quark distributions. This scheme partly shares the properties of the DIS_{γ} scheme to warrant a reasonable behaviour of $F_{2,PL}^{\gamma}$ in the large-x region. In the polarized case it is easy to see that the ansatz of [20] amounts to transforming ΔC_{γ}^n via eq.(13) by

$$\delta \Delta C_{\gamma}^n = -2 \left[-\frac{n-1}{n(n+1)} S_1(n) + \frac{2}{n(n+1)^2} \right] \ , \tag{18}$$

with corresponding changes of $\Delta k_i^{(1)n}$. For completeness we include the result for $g_{1,PL}^{\gamma}$ in this factorization scheme in Fig.1. It turns out that the above mentioned slight violation of positivity does not occur if both the polarized and unpolarized NLO quark densities are defined in this scheme. On the other hand, it becomes obvious that a significant dissimilarity between the LO and NLO results remains, which would demand compensation by sizeably different LO/NLO hadronic inputs[5]. We therefore do not pursue this scheme any further, but will henceforth adopt the DIS_{γ} scheme as introduced above when studying the polarized photon structure beyond the leading order.

For convenience, we provide the relation of the NLO DIS_{γ} and $\overline{\mathrm{MS}}$ photonic parton distributions since it is to be expected that future calculations of NLO corrections to

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[5]Similar features were observed for this scheme in the unpolarized case [18].

polarized cross sections will be carried out within the $\overline{\text{MS}}$ scheme. The Δf^γ in the $\overline{\text{MS}}$ scheme can be obtained by the transformation

$$\Delta f^\gamma_{\overline{\text{MS}}}(x, Q^2) = \Delta f^\gamma_{\text{DIS}_\gamma}(x, Q^2) + \delta \Delta f^\gamma(x, Q^2) \qquad (19)$$

with

$$\delta \Delta q^\gamma(x, Q^2) = \delta \Delta \bar{q}^\gamma(x, Q^2) = -N_C e_q^2 \frac{\alpha}{4\pi} \Delta C_\gamma(x) \,, \quad \delta \Delta g^\gamma(x, Q^2) = 0 \,, \qquad (20)$$

where $\Delta C_\gamma(x)$ is given in eq.(11)[6].

To finish this technical part of the paper, we briefly discuss the so-called NLO 'asymptotic' solution for the spin-dependent parton distributions of the photon, which is obtained by dropping all terms in the full (pointlike) solution which decrease with increasing values of Q^2. In this way all dependence on the input scale and the boundary conditions is eliminated, and one ends up with the unique QCD prediction (see [21, 15, 9, 22] for a discussion of the asymptotic solution in the unpolarized case)

$$\Delta \vec{q}^{\gamma,n}_{PL}(Q^2) = \frac{4\pi}{\alpha_s(Q^2)} \Delta \vec{a}^n + \Delta \vec{b}^n \,, \qquad (21)$$

where

$$\Delta \vec{a}^n = \frac{1}{1 - (2/\beta_0)\Delta \hat{P}^{(0)n}} \frac{\alpha}{2\pi\beta_0} \Delta \vec{k}^{(0)n} \,,$$

$$\Delta \vec{b}^n = -\frac{1}{\Delta \hat{P}^{(0)n}} \left[2 \left(\Delta \hat{P}^{(1)n} - \frac{\beta_1}{2\beta_0} \Delta \hat{P}^{(0)n} \right) \Delta \vec{a}^n + \frac{\alpha}{2\pi} \left(\Delta \vec{k}^{(1)n} - \frac{\beta_1}{2\beta_0} \Delta \vec{k}^{(0)n} \right) \right]. \, (22)$$

The above equations have been written for the singlet case; extension to the non-singlet sector is trivial. The polarized LO asymptotic solution, which was already studied in [23], is entailed in the expressions by dropping all NLO terms, i.e., keeping the $\Delta \vec{a}$ term only. The NLO asymptotic parton densities in (21) are obviously again subject to the factorization convention adopted. For instance, one could choose to work in the $\overline{\text{MS}}$ scheme for which the $\Delta \vec{k}^{(1)n}$ is as given in (7), or again in the DIS_γ where the transformation (14) is to be taken into account. However, unlike the non-asymptotic pointlike solution $g^\gamma_{1,PL}$, the asymptotic prediction for g^γ_1, to be obtained from eqs.(21),(9), is readily seen to be

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[6] Alternatively, of course, one can work directly with the DIS_γ distributions by applying an appropriate transformation [9] to NLO sub-cross sections calculated in the $\overline{\text{MS}}$ scheme for processes involving polarized real photons.

scheme-independent up to terms of $\mathcal{O}(\alpha_s)$, as it has to be. It is also displayed in Fig. 1 for $Q^2 = 20\,\mathrm{GeV}^2$. The practical utility of the asymptotic solution is very limited, since it only applies at very large Q^2 and x. Furthermore, the determinants of the denominators $1 - (2/\beta_0)\Delta\hat{P}^{(0)n}$ and $\Delta\hat{P}^{(0)n}$ in (22) can vanish, causing completely unphysical poles of the asymptotic solution which are *not* present in the full solution where subleading ('non-asymptotic') terms regulate such pole terms. These obvious defects of the asymptotic solution are well-known from the unpolarized case [15, 9] and need not be discussed again in detail. We only mention that the position of the poles in Mellin-n space can differ from the unpolarized case due to the in general different splitting functions involved: For $N_f = 3$ flavors the determinant of $1 - (2/\beta_0)\Delta\hat{P}^{(0)n}$ vanishes for $n = 0.2903$ in the non-singlet case[7] and for $n = 0.3673$ and $n = 1$ in the singlet case. It turns out, however, that the pole at $n = 1$ is cancelled twice by terms in the numerator, such that the LO asymptotic solution has a vanishing first moment. The situation becomes worse at NLO, where the poles arising from $1/\Delta\hat{P}^{(0)n}$ have to be taken into account. Since the first moment of $\Delta P_{qq}^{(0)}$ vanishes, the non-singlet solution has a potential pole at $n = 1$. As in LO, it is cancelled by terms from the numerator, but this time the result remains *finite* at $n = 1$, such that the NS part of the asymptotic solution has a non-vanishing first moment beyond LO. In the singlet sector, the determinant of $\Delta\hat{P}^{(0)n}$ develops a zero at $n = 1.5723$ ($N_f = 3$). This implies that the singlet asymptotic solution will rise as $\approx x^{-1.57}$ as $x \to 0$, i.e., the polarized NLO asymptotic photonic parton densities, as well as the asymptotic result for g_1^γ, will not be integrable anymore. This clearly underlines that the asymptotic solution can in general not be regarded as a reliable or realistic estimate for the polarized photon structure.

To study more quantitatively the influence of the QCD corrections we extend a previous LO analysis of the polarized photon structure within the radiative parton model [14, 12] to NLO in the DIS_γ factorization scheme as described above. As pointed out above, the main advantage of the DIS_γ scheme is [13] that an optimal perturbative stability is achieved for the pointlike part of the photonic structure functions F_2^γ and g_1^γ, implying that no additional 'fine-tuned' input is required in NLO. One thus expects that

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[7]Needless to say that in this case $\Delta\hat{P}^{(0)n} \to \Delta P_{qq}^{(0)n} = P_{qq}^{(0)n}$.

the hadronic inputs in LO and NLO will differ by just the small amounts known from similar analyses of nucleon structure functions (see, e.g., [6], and [5] for the polarized case), and that beyond the LO the DIS$_\gamma$ scheme is the most likely scheme in which a *pure VMD* hadronic input can be successfully implemented. In fact, such a result was found in the unpolarized case in [13], where, starting the evolution from a low input scale, it was shown that rather similar hadronic VMD-inputs were sufficient in LO and NLO to describe existing data for F_2^γ at larger Q^2 accurately. Since nothing is known experimentally about the parton structure of vector mesons, the parton densities of the neutral pion as determined in a previous study [24] were used instead which are expected not to be too dissimilar from those of, e.g., the ρ. Unfortunately, such a procedure is obviously impossible for determining the VMD input distributions $\Delta f^\gamma(x, \mu^2)$ for the *polarized* photon. Therefore, to obtain a realistic estimate for the theoretical uncertainties in the polarized photon structure functions coming from the unknown hadronic input, two very different scenarios were considered in [14, 12]: For the first ('maximal scenario') the input was characterized by

$$\Delta f_{had}^\gamma(x, \mu^2) = f_{had}^\gamma(x, \mu^2) \tag{23}$$

whereas the other extreme input ('minimal scenario') was defined by

$$\Delta f_{had}^\gamma(x, \mu^2) = 0 \tag{24}$$

with $\mu^2 = \mu_{LO}^2 = 0.25\,\mathrm{GeV}^2$ and the unpolarized LO distributions $f_{had}^\gamma(x, \mu^2) \equiv f_{had,LO}^\gamma(x, \mu_{LO}^2)$ taken from [13]. We will closely follow this approach and thus in NLO (DIS$_\gamma$) take $\mu^2 = \mu_{NLO}^2 = 0.3\,\mathrm{GeV}^2$ and the unpolarized NLO densities $f_{had}^\gamma(x, \mu^2) = f_{had,NLO}^\gamma(x, \mu_{NLO}^2)$ from [13] in eqs. (23) and (24) to define the two extreme scenarios. We mention at this point that a sum rule expressing the vanishing of the first moment of the polarized photon structure function g_1^γ was derived from current conservation in [25], which we could use to further restrict the range of allowed VMD inputs. The sum rule can be realized in LO and NLO ($\overline{\mathrm{MS}}$ or DIS$_\gamma$) by demanding

$$\Delta q_{had}^{\gamma, n=1}(\mu^2) = 0 \quad , \tag{25}$$

i.e., a vanishing first moment of the photonic quark densities at the input scale. Inspecting the relevant LO and NLO evolution kernels and coefficient functions for $n = 1$, in

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particular the expressions for the $\Delta k_i^{(1)n}$ in (7) and (14), one finds that the sum rule $g_1^{\gamma,n=1}(Q^2) = 0$ is then maintained for *all* Q^2 even beyond the LO[8]. Of the two extreme hadronic inputs introduced above only the 'minimal' one (eq.(24)) satisfies (25). On the other hand, we are interested only in the region of, say, $x > 0.01$ here, such that for the 'maximal' scenario (23) the current conservation constraints at the input scale could well be implemented by contributions from smaller x which do not affect, of course, the evolutions at larger x. In addition to this, the first moment of the polarized photonic gluon distribution remains completely unconstrained by current conservation considerations. Rather than artificially enforcing the vanishing of the first moment of the $\Delta q_{had}^{\gamma}(x, \mu^2)$ in the 'maximal' scenario (see [14]), we therefore stick to the two extreme scenarios as introduced above.

This fully specifies our polarized photonic NLO (DIS$_\gamma$) distributions $\Delta f^\gamma(x, Q^2)$ for all $Q^2 \geq \mu^2$. The values for the QCD scale parameter $\Lambda_{\overline{\mathrm{MS}}}$ in NLO, appearing in eq.(4) and used in the evolution equations, are also taken from [13], i.e.,

$$\Lambda_{NLO}^{(3,4,5)} = 248,\ 200,\ 131 \ \mathrm{MeV} \ . \tag{26}$$

We adopt all threshold conventions as in [13] and our LO analysis [14, 12].

In Fig.2 we compare our LO [14, 12] and NLO (DIS$_\gamma$) distributions $x\Delta u^\gamma/\alpha$, $x\Delta g^\gamma/\alpha$ for the two extreme scenarios at $Q^2 = 10\,\mathrm{GeV}^2$. As can be seen, the NLO distributions in the DIS$_\gamma$ scheme are very similar to the LO ones. Fig.3 shows the photonic structure function $x g_1^\gamma/\alpha$ in LO and NLO as calculated according to eq.(9). Very satisfactory perturbative stability is found. The result is presented for $N_f = 3$ flavors, i.e., we have not included the charm contribution to g_1^γ which could be calculated via the polarized 'direct' fusion subprocess $\gamma^*\gamma \to c\bar{c}$ and the (small) 'resolved' process $\gamma^* g \to c\bar{c}$ in which the polarized photonic gluon distribution takes part [12]. The charm contribution is immaterial for our more illustrative purposes.

To summarize, we have provided all ingredients for a NLO analysis of the spin-dependent parton distributions of the photon and of its polarized structure function g_1^γ.

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[8]As mentioned above, this is no longer true for the NLO asymptotic solution.

We have shown that g_1^γ suffers from the same perturbative instability problems as the corresponding unpolarized structure function F_2^γ in the $\overline{\text{MS}}$ scheme which hampers a straightforward NLO analysis. As we have demonstrated, it is therefore recommendable to work in a 'polarized version' of the DIS_γ factorization scheme originally introduced in the unpolarized case in order to circumvent such problems. We have finally presented two extreme sets of polarized photonic NLO parton distributions $\Delta f^\gamma(x, Q^2)$.

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Figure Captions

Fig.1 The 'pointlike' part of xg_1^γ/α (see eq.(12)) in LO and NLO for the $\overline{\text{MS}}$ and the DIS$_\gamma$ factorization schemes. Also shown is the result obtained when extending the factorization scheme of [20] to the polarized case ('AFG', see text). The toy input scale $\mu = 1$ GeV, the QCD scale parameter $\Lambda = 200$ MeV and $N_f = 3$ flavors have been used. For illustration the NLO asymptotic solution as obtained from eqs.(21), (22) is included in the lower part for $Q^2 = 20\,\text{GeV}^2$.

Fig.2 Predictions for the NLO (DIS$_\gamma$) polarized photonic parton densities for the 'maximal' and 'minimal' inputs of eqs.(23) and (24), respectively. For comparison we also show the corresponding LO results of [14, 12].

Fig.3 NLO predictions for the spin-dependent photon structure function g_1^γ for the 'maximal' and 'minimal' inputs of eqs.(23) and (24), respectively. The results shown correspond to $N_f = 3$ flavors. For comparison we also present the respective LO predictions of [14, 12].
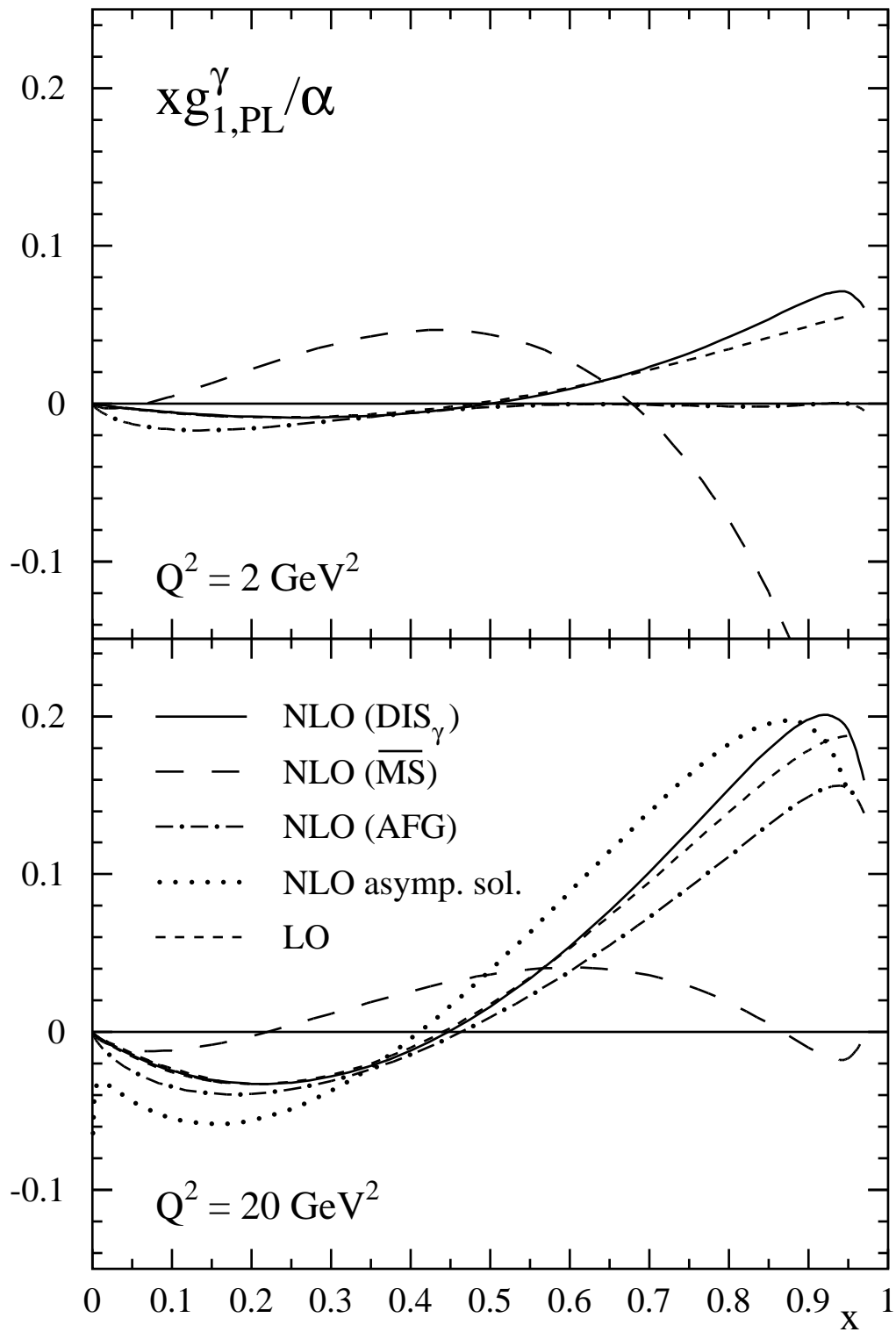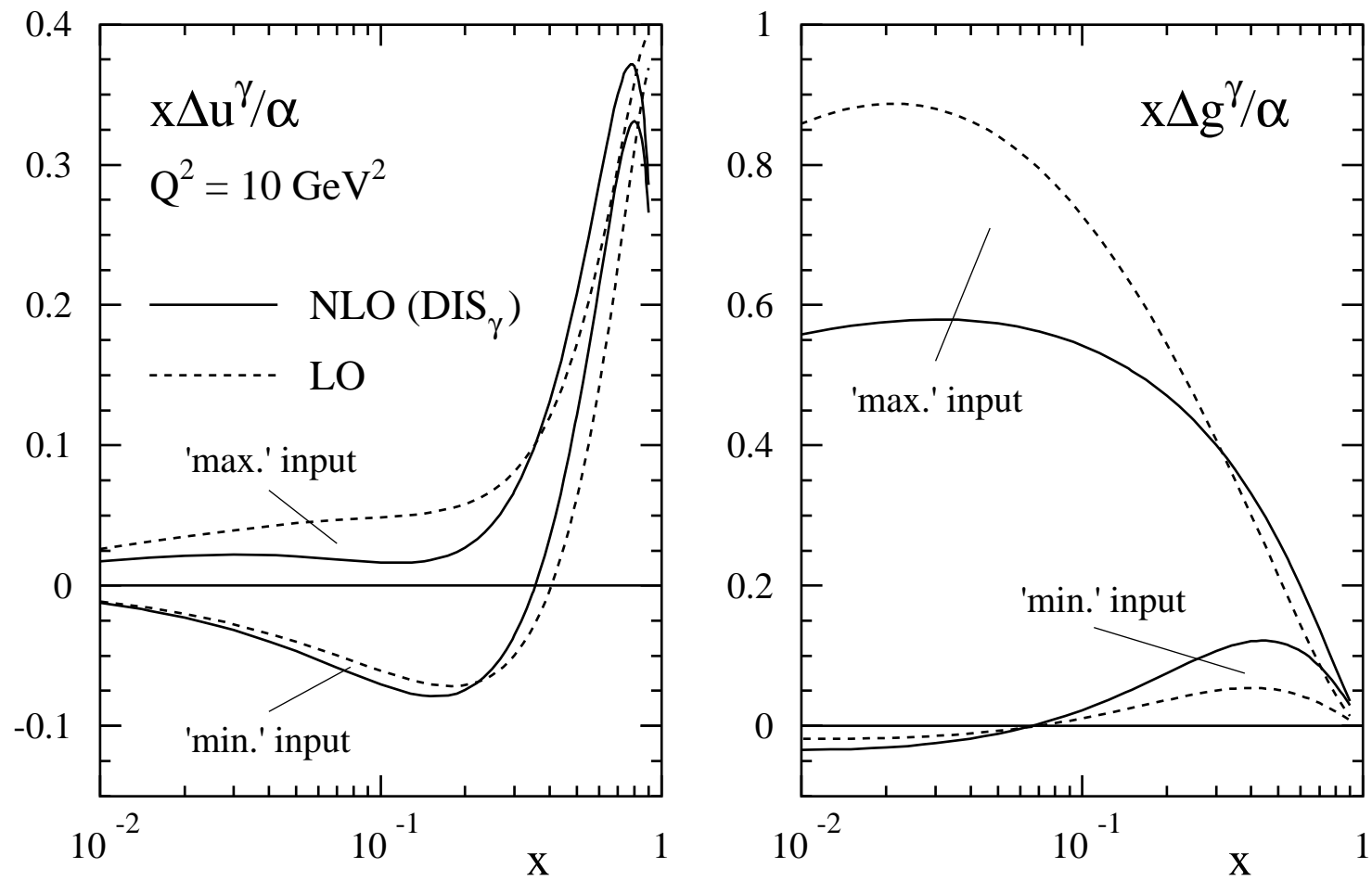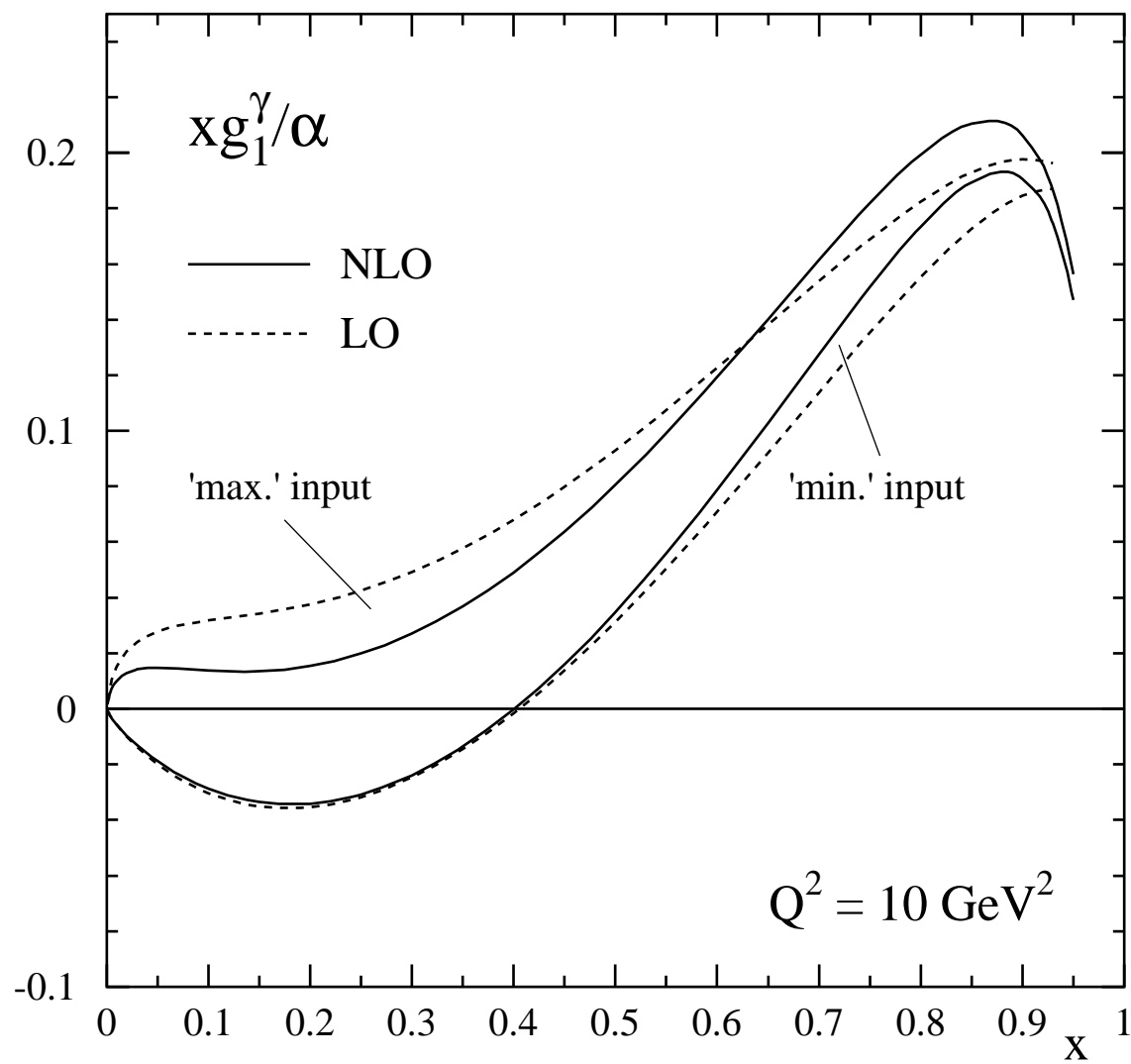
Fig. 1

Fig. 2

Fig. 3